Hamitlon-Jacobi equations on networks

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Abstract

I will provide a survey of recent results on the study of Hamilton-Jacobi equations on networks. Note that the problem is not a straightforward generalization of the Euclidean setting (the one thoroughly studied, e.g., in the Bardi-Capuzzo Dolcetta book [2]) because of the presence of the vertices.

With the aim of extending the notion of viscosity solution to networks, three different approaches have been recently proposed ([1], [3], [4]). All the three papers lead to a mathematically convenient theory (i.e. a theory for which existence, uniqueness and stability hold). But since they are motivated by different applications (respectively, a control problem constrained to a network in [1], the study of traffic flow at a junction in [3] and Eikonal equations and distance functions on networks in [4]), they differ for the assumptions made on the Hamiltonian and especially for the definitions of viscosity solution at the vertices (while inside the edges all the definitions coincide with the classical one).

In this talk I will mainly concentrate on the results in [4], but I will also try to point out similarities and differences with the other two approaches.

A preliminary plan of the talk is the following:
In the 1^{st} lecture, I will briefly review the linear theory of differential equations on networks. In the 2^{nd} lecture I will describe the notion of viscosity solution for Hamilton-Jacobi equation on networks in [4]. The 3^{rd} lecture is devoted to the comparison among the various notions of viscosity solution. In the last 45 minutes-lecture I will switch to a different, even if connected argument, namely eikonal equation on Sierpinski gasket and pcf fractal sets.

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References

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[3] C. Imbert, R. Monneau and H. Zidani, *A Hamilton-Jacobi approach to junction problems and application to traffic flows*, ESAIM Control Optim. Calc. Var. 19 (2013), 129-166.

[4] D. Schieborn and F. Camilli, *Viscosity solutions of Eikonal equations on topological network*, Calc. Var. Partial Differential Equations. 46 (2013), 671–686.