

Discontinuous Galerkin method for Hamilton-Jacobi equations and front propagation with obstacles

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Abstract

In this talk we will first describe a discontinuous Galerkin (DG) method for solving Hamilton-Jacobi equations, including those for front propagation problems. This method solves the Hamilton-Jacobi equations directly, without first converting them to conservation law systems, can be proved to converge optimally in L^2 for smooth solutions, and perform nicely for viscosity solutions with singularities.

We then extend the DG method to front propagation problems in the presence of obstacles. We follow the formulation of Bokanowski et al. leading to a level set formulation driven by $\min(u_t + H(x, \nabla u), u - g(x)) = 0$, where $g(x)$ is an obstacle function. The DG scheme is motivated by the variational formulation when the Hamiltonian H is a linear function of ∇u , corresponding to linear convection problems in presence of obstacles. The scheme is then generalized to nonlinear equations, resulting in an explicit form which is very efficient in implementation. Stability analysis are performed for the linear case with Euler forward, a second and third order SSP Runge-Kutta time discretization, and convergence is proved for the linear case with Lipschitz continuous and piecewise smooth data. Numerical examples are provided to demonstrate the robustness of the method. Finally, a narrow band approach is considered in order to reduce the computational cost. This is a joint work with Yingda Cheng (the design of the scheme), Tao Xiong (error estimates for smooth solutions), and Olivier Bokanowski and Yingda Cheng (front propagation without and with obstacles).