Traffic flow on networks: modeling, optimization, and Nash equilibria

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Abstract

This short course will focus on conservation law models for traffic flow on a network of roads. An outline of the topics covered is as follows:

1. A brief review the basic theory of scalar conservation laws: shocks, Rankine-Hugoniot equations and entropy conditions. Variational analysis of solutions and their dependence on the initial data. The Lighthill-Whitham conservation law model for vehicle flow on a single road.

2. Boundary conditions modeling traffic flow at road junctions. Construction of Riemann solvers with good stability properties. The Cauchy problem for traffic flow on a network of roads. Existence and uniqueness of solutions.

3. Cost functionals involving the travel time and a penalty function for late arrival. Global optimization problems, for several groups of drivers on a network of roads. Existence and uniqueness results. Necessary conditions for optimality.

4. Definition and properties of Nash equilibrium solutions. Existence, uniqueness, and characterization of Nash equilibria.

The final part of the course will be devoted to a discussion of research directions and open problems.