Hamiltion-Jacobi equations on networks as limits of singularly perturbed problems in optimal control: dimension reduction

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Abstract

We consider a family of open star-shaped domains $\Omega^\varepsilon$ of $\mathbb{R}^2$. Roughly speaking, $\Omega^\varepsilon$ is made of a finite number of non intersecting semi-infinite strips of thickness $\varepsilon$ and of a central region whose diameter is of the order of $\varepsilon$, that may be called the junction. When the thickness $\varepsilon$ tends to 0, the domains $\Omega^\varepsilon$ tend to a union of half-lines sharing an endpoint $O$. This set is termed network. We study infinite horizon optimal control problems in which the state is constrained to remain in $\Omega^\varepsilon$. In the above mentioned strips the running cost may have a fast variation w.r.t. the transverse coordinate. We pass to the limit as the parameter $\varepsilon$ tends to zero, and prove that the value function tends to the solution of a Hamilton-Jacobi equation on the network, which may also be related to an optimal control problem. One difficulty is to find the transmission condition at the junction node $O$ in the limit problem. For passing to the limit, we use the method of the perturbed test-functions of Evans, which requires constructing suitable correctors. This is another difficulty since the domain is unbounded.

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References
