Hamilton-Jacobi equations on networks as limits of singularly perturbed problems in optimal control: dimension reduction

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Abstract

We consider a family of open star-shaped domains Ω^ϵ of $I\!R^2$. Roughly speaking, Ω^ϵ is made of a finite number of non intersecting semi-infinite strips of thickness ϵ and of a central region whose diameter is of the order of ϵ, that may be called the junction. When the thickness ϵ tends to 0, the domains Ω^ϵ tend to a union of half-lines sharing an endpoint O. This set is termed *network*. We study infinite horizon optimal control problems in which the state is constrained to remain in $\overline{\Omega^\epsilon}$. In the above mentioned strips the running cost may have a fast variation w.r.t. the transverse coordinate. We pass to the limit as the parameter ϵ tends to zero, and prove that the value function tends to the solution of a Hamilton-Jacobi equation on the network, which may also be related to an optimal control problem. One difficulty is to find the transmission condition at the junction node O in the limit problem. For passing to the limit, we use the method of the perturbed test-functions of Evans, which requires constructing suitable correctors. This is another difficulty since the domain is unbounded.

Acknowledgments The authors were partially funded by the ANR project ANR-12-BS01-0008-01. The first author was partially funded by the ANR project ANR-12-MONU-0013.

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References

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