Indefinite Linear MPC and Approximated Economic MPC for Nonlinear Systems

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joint work with *Sébastien Gros,* and *Moritz Diehl*

Abstract

Economic Model Predictive Control (MPC) has recently gained popularity, as it directly optimizes a given performance index, as opposed to tracking MPC, which minimizes the deviation from a given reference. The main advantage of economic MPC over tracking MPC becomes obvious in transients, when the system is steered to steady state so as to minimize the given performance index. Stability theory for economic MPC is, however, not as mature as for the tracking case. Important contributions have been given in [1, 2, 4, 6], where the concepts of rotated cost and strict dissipativity play a fundamental role for the stability proofs.

In the nonlinear case, the strict dissipativity condition can be extremely hard to check, thus making it difficult to ensure stability. The analysis becomes simpler in the linear quadratic possibly indefinite case and the conditions that ensure stability for the linear quadratic regulator (LQR) have been studied in [3, 5].

This talk presents the main results of [7], where it has been shown that any stabilizing indefinite LQR or MPC scheme can be reformulated as a positive definite one. First, the conditions for the existence of a stabilizing indefinite LQR are given. Then, two transformations under which the LQR is unchanged are introduced. By combining these two transformations, the problem is reformulated in a form for which it is easy to prove the existence of a positive definite equivalent formulation using Lyapunov arguments. The results are finally extended to the case of a finite horizon and a terminal cost different from the LQR cost-to-go.

The talk concludes by proposing a tracking nonlinear MPC formulation that approximately solves economic nonlinear MPC. The derivation is based on the results for the linear quadratic indefinite case and therefore relies on a local stabilizability assumption. The interest of such an approximated scheme is twofold as a) it has the stability properties of tracking schemes and b) it allowes the deployment of efficient numerical methods.

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