Quick reachability and proper extension of problems with unbounded controls

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Abstract

We consider control systems of the form

$$\text{(1)} \qquad \begin{cases} \dot{x} = f(t,x,u,v) + \sum_{\alpha=1}^{m} g_\alpha(x,u)\dot{u}_\alpha, \quad t \in [a,b], \\ x(a) = \bar{x} \end{cases}$$

where the state x belongs to $\mathbb{R}^n$, the control $v : [a,b] \to V \subset \mathbb{R}^d$ is an ordinary bounded control while, due to the presence of the derivative $\dot{u}$, $u : [a,b] \to U \subseteq \mathbb{R}^m$ may be regarded as an *impulsive control*. The notion of *limit solution* for (1), which allows for everywhere defined measurable maps, is here adopted. For a problem in the Mayer form with final constraints of the type $(x(b), u(b)) \in \mathcal{S}$, we investigate the question whether this notion of solution provides a *proper extension* of the standard problem with absolutely controls u, i.e. if the subset of trajectories of the latter is dense (in a suitable sense) in the set of trajectories of the former and the two infimum values do coincide. Different approximation results are provided, depending on the regularity of the boundary of the target $\mathcal{S}$ and of "fast reachability" conditions.

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