Viscosity methods for multiscale financial models with stochastic volatility

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Abstract

Financial models describe the evolution of the price of an asset by a stochastic differential equation of the form

$$dX_t = f(X_t)dt + X_t\sigma dW_t,$$

where W_t is Brownian motion. In optimal portfolio optimisation the wealth evolves in a similar way with f and σ depending also on an admissible control process u_t. Models with stochastic volatility assume that $\sigma = \sigma(Y_t)$ depends on another stochastic process Y_t. Typically Y_t is a mean-reverting diffusion process driven by a Brownian motion correlated to W_t. About 15 years ago it was argued by various authors including Fouque, Papanicolaou, and Sircar [4] that the volatility process Y_t evolves at a faster time scale than the assets. This leads to models where $Y_t = Y_t^\epsilon$ depends on a small parameter $\epsilon > 0$ that are studied by methods of asymptotic analysis. A number of issues have been studied for such multiscale models. In [1] methods from the theory of viscosity solutions were first used to prove rigorously the convergence as $\epsilon \to 0$ in models such as Merton portfolio optimisation for a suitably scaled Ornstein-Uhlenbeck volatility process Y_t^ϵ. The analysis was based on the Hamilton-Jacobi-Bellman satisfied by the optimal expected wealth of the portfolio, where the terms involving ϵ appear as singular perturbations.

In this talk we present results on two problems related to this framework. The first is a multiscale model where the volatility process Y_t^ϵ follows an

Ornstein-Uhlenbeck equation driven by a Lévy process with jumps, as proposed by the important paper of Barndorff-Nielsen and Shephard [2]. Here the H-J-B equation is integrodifferential. We get results comparable to the case of Gaussian noise by means of the ergodicity properties of the volatility process and viscosity methods for the H-J-B equation.

The second problem is inspired by [3] and concerns models with Gaussian noise, as in [4, 1], where the short maturity limit is analysed. Here there are two small parameters, ϵ and δ, and we look for a large deviation principle, as in [3]. We discover three possible regimes that depend on the reciprocal speed of the parameters, and find rigorously the limit in all three cases.

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References

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