

Parametric Sensitivity Analysis and Real-Time Optimal Control using TransWORHP

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Abstract

Algorithms for the numerical solution of discretized optimal control problems are already used successfully in many areas of application, e.g. robotics, aerospace engineering, chemistry, or automotive industry. However, to improve the industrial applicability in case of perturbations, strategies for real-time solutions are necessary.

We state the following perturbed optimal control problem depending on a parameter $p \in \mathbb{R}^{n_p}$:

$$\begin{aligned} \min_{x,u,t_f} \quad & \Psi(x(t_f), p) + \int_0^{t_f} f_0(x(t), u(t), p, t) dt \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t), u(t), p, t), \quad t \in [0; t_f] \\ & \omega(x(0), x(t_f), p) = 0 \\ & g(x(t), u(t), p) \leq 0, \quad t \in [0; t_f] \end{aligned}$$

For a fixed parameter $p = p_0$ this problem can be interpreted as unperturbed. To solve it, a direct method replaces the infinite dimensional function space of state variables $x : [0; t_f] \rightarrow \mathbb{R}^n$ and control variables $u : [0; t_f] \rightarrow \mathbb{R}^m$ by the finite dimensional space of real numbers. Hence only $x_i \approx x(t_i)$, $u_i \approx u(t_i)$ are considered for discrete time points $0 = t_1 < t_2 < \dots < t_l = t_f$.

By applying integration schemes like the Trapezoidal Method or Hermite Simpson, our transcription method TransWORHP reformulates an optimal

control problem to a nonlinear optimisation problem [1]:

$$\begin{aligned} \min_z \quad & F(z, p) \\ \text{s.t.} \quad & g(z, p) \leq 0 \\ & h(z, p) = 0 \end{aligned}$$

Here, the dimension of $z \in \mathbb{R}^N$ containing all discretized state and control variables might become very large. Using the SQP solver WORHP (We Optimize Really Huge Problems) the special structure of the discretized optimal control problem can be exploited to solve it even for millions of discretized variables and constraints [2].

For an optimal solution, the recent module WORHP Zen provides parametric sensitivity differentials, e.g. $\frac{dz}{dp}(p_0)$. These values describe how the unperturbed optimal solution would change if the parameter vector p is modified slightly [3].

We will discuss methods, how these *off-line* values can also be used for a fast *on-line* approximation of the perturbed solution. The cost-efficiency and robustness of these iterative methods will be illustrated by applied examples using tools available in TransWORHP.

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References

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