On the singularities of minimum time function for normal linear control systems

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Abstract

Consider the linear control system $x' = Ax + bu$ in $\mathbb{R}^N$ with A and b satisfying the Kalman rank condition

$$\mathrm{rk}[b, Ab, \dots, A^{N-1}b] = N,$$

where here for simplicity the control is assumed to be single-input, with $u \in [-1, 1]$. Results stating that

a) the minimum time function T to reach the origin is Hölder with exponent $1/N$ in the reachable set (which contains a neighborhood of 0);

b) the optimal control is unique, bang-bang with an upper bound on the number of switchings;

c) the reachable sets at every time are strictly convex;

are classical from the early stages of control theory. Simple examples ($\ddot{x} = u$, $\ddot{x} + x = u$) show that T is **never** everywhere differentiable, and even Lipschitz, in any deleted neighborhood of the origin. The talk will be devoted to describing the following results:

1) a quantitative estimate on the modulus of strict convexity of reachable sets;

2) T is differentiable in an open set with full measure;

3) more precisely, T is analytic out of a closed set C which is a countable union of Lipschitz graphs of $N-1$ variables;

4) some information on the exceptional set C can also be provided: in particular, the set where T is not Lipschitz is fully described.

Methods of nonsmooth analysis and of geometric measure theory are used. As an application, we prove that T is (locally in the reachable set) is Sobolev and is a SBV function, namely T has bounded variation and the Cantor part of its derivative vanishes.
The analysis of the singular set of value functions is well studied. This talk focuses on singularities due to switchings and to higher order controllability.

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References

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