Optimal control of stochastic processes via probability density distribution function control

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Abstract

A new framework that uses the probability density function (PDF) for the optimal control of stochastic processes, as representative of the stochastic state, is presented. This framework is based on the problem of minimizing a cost function with the Kolmogorov-Fokker-Planck-type (KFP) equations as constraint, governing the time evolution of the PDF. In particular if the control objective is formulated for tracking purpose of a given PDFs trajectory, then the corresponding optimal control problem results as a sequence of open-loop optimality systems in a nonlinear model predictive control strategy. The optimality system consists of forward (KFPE) and backward partial differential equations, and an optimality equation, whose solution solves the problem to find a controller that minimizes the cost function within a time interval under the constraint provided by the KFP equation. The optimality system is solved numerically by using a positive and conservative discretization scheme for the forward equation. The effectiveness of this innovative framework is shown by using models from biology, physics, and finance.

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References

[1] M. Annunziato, A. Borzì, Fokker-Planck-based control of a two level open quantum system, Math. Models and Meth. in Appl. Sci. Vol. 23 (2013) No. 11, pp. 2039-2064

[2] M. Annunziato, A. Borzì, A Fokker-Planck control framework for multidimensional stochastic processes, Journal of Comp. and Appl. Math., Vol. 237 (2013) No. 1, pp. 487-507

[3] M. Annunziato, A. Borzì, Optimal control of probability density functions of stochastic processes, Mathematical Modelling and Analysis, Vol. 15 (2010) No. 4, pp. 393-407