The Minimum Principle for State-Constrained Optimal Control Problems with Time Delays

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joint work with *Laurenz Göllmann and Richard Vinter*

Abstract

We study the following state-constrained optimal control problem with multiple time delays:

$$\text{Minimize} \quad J(u,x) = g(x(T))$$

over measurable functions $u : [0,T] \to \mathbb{R}^m$ and arcs $x \in W^{1,1}([0,T],\mathbb{R}^n)$ satisfying

$$\begin{aligned}
&\dot{x}(t) = f(t, x(t-d_0), \ldots, x(t-d_k), u(t-d_0), \ldots, u(t-d_k)) \text{ a.e. } t \in [0,T],\\
&x(t) = x_0(t) \quad \forall\, t \in [-d_k, 0], \quad \psi(x(T)) = 0,\\
&u(t) = u_0(t) \quad \forall\, t \in [-d_k, 0), \quad u(t) \in U \subset \mathbb{R}^m \text{ a.e. } t \in [0,T],\\
&S(t, x(t-d_0), \ldots, x(t-d_k)) \le 0 \text{ a.e. } t \in [0,T],
\end{aligned}$$

the data for which comprise an interval $[0,T]$, constant time delays $0 = d_0 < d_1 < \ldots < d_k$, $(k \ge 1)$, Lipschitz functions $g : \mathbb{R}^n \to \mathbb{R}$, $f : [0,T] \times \mathbb{R}^{(k+1)\cdot n} \times \mathbb{R}^{(k+1)\cdot m} \to \mathbb{R}^n$, $\psi : \mathbb{R}^n \to \mathbb{R}^q$ $(0 \le q \le n)$, and $S : [0,T] \times \mathbb{R}^{(k+1)n} \to \mathbb{R}$ defining the state constraint. The initial functions satisfy $x_0(\cdot) \in L^\infty([0,T],\mathbb{R}^n)$ and $u_0(\cdot) \in L^\infty([0,T],\mathbb{R}^m)$.

We assume that the delays *commensurate* which means that there exists $\Delta > 0$ and integers $0 = n_0 < n_1 < \ldots < n_k$ with $d_j = \Delta \cdot n_j$ for $j = 0, 1, \ldots, k$. The transformation technique proposed by Guinn [3] then permits the replacement of the delayed optimal control problem by a non-delayed control problem on the interval $[0,\Delta]$ to which we may apply the Minimum Principle for non-delayed problems with state constraints; cf. e.g. Maurer

[4], Vinter [6]. The backward transformation of the adjoint variables and multipliers furnishes a Minimum Principle for the delayed control problem; cf. also Göllmann, Maurer [2].

We briefly sketch numerical discretization and optimization methods which are similar to those in [2]. The theory and numerical approach are illustrated by two examples in chemical engineering, resp., in biomedicine that extend optimal control models in Dadebo, Luus [1], resp., Stengel et al. [5].

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