

# On the Optimization of Riemann-Stieltjes-Control-Systems with Application in Vehicle Dynamics

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## Abstract

Optimizing discontinuous systems is a field of research with many unsolved problems. In this work, we focus on a method to model dynamic multibody control systems subject to state dependent frictional impacts using Riemann-Stieltjes-Integrals. As the impulsive character of the system is based on the state only, the controls exclusively enter the continuous Riemannian part of the dynamics. The system equations are combined with a set of impact conditions, which are introduced to calculate the jumps of generalized velocities at contact using a formulation with energetically consistent generalized coefficients-of-restitution. These impact conditions are a reformulation of a complementarity problem with additional constraints on the post-impact velocities. Based on this model of differential equations with state discontinuities, a gradient based optimization routine is presented making use of the bounded variation assumption in the velocity components. The velocity jumps are computed by evaluating the frictional model as a minor optimization problem for the coefficients-of-restitution in tangential direction. The theoretical method has shown to be applicable for different optimization problems with continuous state dependent cost functionals. The method is illustrated in a numerical example, where comfort and handling characteristics for vehicle models are optimized due to road irregularities. Last, we draw a comparison between results derived by this method and a commercial library for tyre-road interaction forces.

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