Bang–bang trajectories with a double switching time in the minimum time problem

Laura Poggiolini
Università degli Studi di Firenze
laura.poggiolini@unifi.it

joint work with *Marco Spadini*

Abstract

We consider the minimum time problem with fixed end-points on a finite dimensional manifold M in the case when the dynamics is affine with respect to the control and the control set is a box in $\mathbb{R}^m$. Namely, we consider the following optimal control problem:

$$T \to \min, \tag{1a}$$

$$\dot{\xi}(t) = f_0(\xi(t)) + \sum_{s=1}^{m} u_s(t) f_s(\xi(t)), \tag{1b}$$

$$\xi(0) = x_0, \qquad \xi(T) = x_f, \tag{1c}$$

$$|u_s(t)| \leq 1 \quad s = 1, 2, \ldots, m \qquad \text{a.e. t} \in [0, \text{T}]. \tag{1d}$$

For such problem , we say that (T, ξ, u) is an *admissible triple* if $T > 0$ and the couple $(\xi, u) \in W^{1,\infty}([0,T], M) \times L^\infty([0,T], \mathbb{R}^m)$ satisfies (1b), (1c) and (1d). We assume we are given a reference triple $\left(\widehat{T}, \widehat{\xi}, \widehat{u}\right)$ which satisfies Pontryagin Maximum Principle (PMP) with an associated covector $\widehat{\lambda}$, and where the reference control $\widehat{u}$ is a regular bang-bang control with a double switching time $\widehat{\tau}$ and a finite number of simple switching times.

We are interested in giving sufficient conditions for *strong local optimality* of the triplet, according to the following definition:

The trajectory $\widehat{\xi}$ is a state-local minimiser if there is a neighborhood $\mathcal{U}$ of its range in M such that $\widehat{\xi}$ is a minimiser among the admissible trajectories whose range is in $\mathcal{U}$.

Our sufficient conditions are given using Hamiltonian methods, following the same lines of the case where only simple switches occur (see [1]) but here the presence of a double switch makes the proof of the local invertibility of the maximised Hamiltonian flow much more tricky. To keep the notation to the minimum in this talk I will consider the case where $m = 2$ and only the double switch occurs. In fact this case already contains most of the mathematical difficulties of the proof. Namely, the presence of a double switch gives rise to a piecewise C^1 (PC^1) maximised Hamiltonian flow where the number of *smoooth pieces* around $\widehat{\lambda}(\widehat{\tau})$ is five, thus requiring a non trivial proof of the local invertibility of such flow, see [2].

More precisely, the definition of PC^1 maps is the following: *Given two finite dimensional manifolds N_1 and N_2, we say that a function $\gamma \colon N_1 \to N_2$ is a continuous selection of C^1 functions if γ is continuous and there exists a finite number of C^1 functions $\gamma_1, \ldots, \gamma_k$ from N_1 in N_2 such that the* active index set $I := \{i \in \{1, 2, \ldots, k\} \colon \gamma(x) = \gamma_i(x)\}$ *is nonempty for each $x \in N_1$. The functions γ_i's are called* selection functions *of γ.*
A continuous function γ is called a PC^1 function if at every point $x \in N_1$ there exists a neighborhood V such that the restriction of γ to V is a continuous selection of C^1 functions.

*

References

[1] Laura Poggiolini and Gianna Stefani. State-local optimality of a bang-bang trajectory: a Hamiltonian approach. *Systems & Control Letters*, 53:269–279, 2004.

[2] Laura Poggiolini and Marco Spadini. Local inversion of planar maps with nice nondifferentiability structure. *Advanced Nonlinear Studies*, 13:411–430, 2013.