New Contributions to Theory and Numerics for State-Constrained Elliptic Optimal Control Problems

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Based on different reformulations of state-constrained elliptic optimal control problems with distributed control and a hypothesis on the structure of the active set, new necessary conditions are obtained which exhibit higher regularity of the multiplier associated with the state constraint. Moreover, we obtain also new jump and sign conditions. Measures are no longer an issue, so that regularization techniques become superfluous. Finally, the method can be fully described in function spaces which is an essential element to obtain a mesh-independent numerical method.

The new approach mimics the well-known Bryson-Denham-Dreyfus indirect adjoining method which is the preferred ansatz in solving state-constrained optimal control problems with ordinary differential equations numerically. However, in the context of PDE constrained optimization this approach turns out to be extremely involved.

Mathematically the reformulations lead to a new kind of set optimal control problem, where the active set of the state constraint, resp. the interface between the inactive and the active set are to be determined as part of the solution as the switching points in the multipoint boundary value problems based on the first-order necessary conditions for ODE constrained optimal control problems. Various formulations of this type of PDE optimization problem as shape and/or topology as well as bilevel optimization problem or, concerning the numerical solution, as free boundary value problem are discussed.

Moreover, parallels can be drawn to optimization on vector bundels which seems to be essential for the design of an appropriate Newton-type method, since the optimization over sets of admissible active sets have a nonlineare structure. This requires an answer to the question how does a Newton method looks on a nonlinear manifold where no Banach space structure is present.

Numerical results will demonstrate the performance of the new method.

This presentation will open new research directions in PDE constrained optimal control as indicated in the final outlook.