

Numerical methods for optimal control of the wave equation

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Abstract

Numerical methods for solving optimal control problems governed by the wave equation are presented.

The talk consists of two parts. In the first part we consider an open-loop approach and consider non-smooth optimal control problems for the wave equation in which the non-smoothness arises either from constraints on the controls [2] or from the choice of the control costs in the cost functional [1]. To solve these problems we apply a semi-smooth Newton method and analyze its convergence. The theoretical results are confirmed by numerical examples.

In the second part we consider a closed-loop approach. An optimal finite-time horizon feedback control problem for one-dimensional wave equations is presented, see [3]. The feedback law can be derived from the dynamic programming principle and requires to solve the evolutionary Hamilton-Jacobi-Bellman (HJB) equation. Since solving the discretized HJB equation in a high-dimensional finite dimensional space, as it is typically the case if the discretization of the underlying wave equation is based on finite elements, is infeasible, in this approach spectral elements are used. The effect of noise is considered and numerical examples are presented.

This is joint work with Karl Kunisch (Universität Graz), Boris Vexler (TU München), and Hasnaa Zidani (ENSTA ParisTech).

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References

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